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# NAVWEPS REPORT

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ANALYSIS OF THE PUFFS TRACKER WOX-1A AS A GENERAL NONLINEAR FILTER (U)

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ANALYSIS OF THE PUFFS TRACKER WOX-1A  
AS A GENERAL NONLINEAR FILTER

Prepared by:

C. Nicholas Pryor

ABSTRACT: A statistical analysis is made of the PUFFS Target Tracker WOX-1A. This device uses a nonlinear tracking scheme such that the estimated target position is changed in discrete steps of fixed magnitude, dependent only upon the sign of observed error at regular sampling intervals.

The theoretical performance characteristics of the tracker are determined from the statistical analysis. Equations are developed for the output standard deviation of tracker position, the lag in tracker position due to bearing rates, and the equivalent integration time of the tracker as functions of the input standard deviation and the tracker step size. These equations are expressed in a sufficiently general form so that they can be readily applied to other systems. Refinements are also discussed which eliminate the bearing lag problem, and stability of the rate-correction system is investigated.

Finally experimental verification is presented for some of the results, and the qualitative operation of the tracker is discussed as an adaptive system interchanging bandwidth and output noise.

Other applications for nonlinear systems of this type are mentioned, including tracking and post-filtering problems in sonar or radar, and filtering problems in navigation systems.

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6 February 1961

This report is a statistical and information theory analysis of the performance of an automatic target tracker developed at the Naval Ordnance Laboratory in which it is shown that the tracker is a special case of a group of generalized nonlinear filters. This work was funded under Task No. WFO08-04-004-RUDC-3C-150, PUFFS Technical Direction. This report will be of special interest to those concerned with the sonar automatic tracking problem and to specialists in signal processing. However, the theory should also be useful in radar and navigation systems and has possible applications in the fields of communications, IFF and detection.

W. D. COLEMAN  
Captain, USN  
Commander

*Z. I. Slawsky*  
Z. I. SLAWSKY  
By direction

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ANALYSIS OF THE PUFFS TRACKER WOX-1A  
AS A GENERAL NONLINEAR FILTER

## INTRODUCTION

The automatic tracker<sup>1</sup> developed for the PUFFS sonar system is used to follow the position of the correlogram in the PUFFS display, and to smooth out the major fluctuations of the correlogram which occur due to noise in the environment and finite integration time. The operation of the tracker is basically as follows: Each time a correlogram is generated (once every 25 milliseconds) the tracker makes an area measurement to determine whether the center of the correlogram is to the right or to the left of the last tracker estimate of the position. On the basis of this measurement, the tracker then changes its estimate by a fixed amount in the direction toward the observed center of the correlogram. This is the nonlinear feature of the tracker; a linear system would move a distance proportional to the observed error, whereas the tracker makes a correction of constant magnitude regardless of the magnitude of the error.

A post integrator for the PUFFS system has also been proposed in which samples of the observed correlogram are compared with stored estimates of the correlogram at a number of corresponding delay times. Then the estimates are increased or decreased a fixed amount depending on whether the observed sample is larger or smaller than the stored values for the given delay. It may be seen that both these devices are similar in principle and may be generalized into a class of nonlinear filters. Such a filter would repeatedly compare its stored estimate of the input variable with samples of the input and then change the stored value at each comparison by a fixed step size in the direction of the observed input value. The analysis will therefore be in terms of this generalized filter, and the results will apply equally well to the tracker, post integrator, or any other device operating in a similar manner. Although the analysis here is based on a sampled system with quantized output values, the results may be extended to continuous systems with continuous output distributions. The sampled approach was chosen because it affords more direct analysis without discussion of frequency response of the elements, and because it corresponds to the cases of interest in the PUFFS system.

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<sup>1</sup>Munson, Cope, Sabelhaus, "Target Tracker WOX-1A", NAVORD Report 6799 (in preparation).

## THEORY OF OPERATION

One of the first items of interest in discussing the operation of such a filter is the standard deviation of the noise output of the system, when white noise of a known standard deviation is applied to the input. We may determine this by computing the probability of finding the output of the filter in each of its possible output positions or states. If the filter output is in its  $k$ th state so that the output value  $x_0$  of the filter is  $x_0 = \Delta k$ , where  $\Delta$  is the size of the steps and the 0th state is defined to be at  $x_0 = 0$ , then the probability of the input sample being less than this value is by definition  $F(x_0)$  or  $F(\Delta k)$  where  $F(x)$  is the cumulative distribution function of  $x_0$ . This is the probability that the filter will decrease its output value by the step  $\Delta$ . Similarly the filter will increase its output with a probability  $1 - F(\Delta k)$  or the probability that the input sample is larger than  $\Delta k$ . If we assume that the system is in its steady state, then the probability of moving from its  $k$ th state to its  $k + 1$ st state must equal the probability of moving from the  $k + 1$ st state into its  $k$ th state. The probability of moving from  $k$  to  $k + 1$  is equal to the probability of being in the  $k$ th state  $P_k$  times the probability of increasing its value or  $1 - F(\Delta k)$ . The probability of moving from  $k + 1$  to  $k$  is  $P_{k+1}$  times the probability of decreasing its value or  $F(\Delta(k + 1))$ . Equating these we have:

$$P_k (1 - F(\Delta k)) = P_{k+1} F(\Delta(k + 1)) \text{ or } \frac{P_{k+1}}{P_k} = \frac{1 - F(\Delta k)}{F(\Delta(k + 1))} \quad (1)$$

This is the basic equation relating the state probabilities for the filter. By iterating equation (1) we can find the probability of any state  $k$  in terms of  $P_0$  since

$$P_k = P_0 \frac{1 - F(0)}{F(\Delta)} \cdot \frac{1 - F(\Delta)}{F(2\Delta)} \cdots \frac{1 - F(\Delta(k - 1))}{F(\Delta k)}$$

This can be written in the form:

$$P_k = P_0 \frac{\prod_{j=0}^{k-1} (1 - F(\Delta j))}{\prod_{j=1}^k F(\Delta j)} \quad (2)$$

If we assume the input to be white noise having a Gaussian distribution with a mean of zero and a standard deviation  $\sigma_1$  then for values of  $x$  small compared to  $\sigma_1$  the distribution function  $F(x)$  becomes approximately

$$F(x) \cong \frac{1}{2} + \frac{x}{\sqrt{2\pi} \sigma_1} + ( \quad ) x^2 + \cdots$$



Inserting this in equation (2) we obtain

$$P_k = P_0 \frac{\prod_{j=0}^{k-1} \left(1 - \sqrt{\frac{2}{\pi}} \frac{\Delta_j}{\sigma_i}\right)}{\prod_{j=1}^k \left(1 + \sqrt{\frac{2}{\pi}} \frac{\Delta_j}{\sigma_i}\right)} = P_0 \frac{\prod_{j=0}^{k-1} (1 - \alpha_j)}{\prod_{j=1}^k (1 + \alpha_j)} \quad (3)$$

where

$$\alpha = \sqrt{\frac{2}{\pi}} \frac{\Delta}{\sigma_i}$$

since

$$\prod_{j=0}^n (1 + \alpha_j) = \prod_{j=1}^n (1 + \alpha_j) = 1 + \alpha \sum_{j=1}^n j + \binom{n}{2} \alpha^2 + \dots = 1 + \frac{n(n+1)}{2} \alpha + \binom{n}{2} \alpha^2$$

we can evaluate equation (3) as:

$$P_k \cong P_0 \frac{1 - \frac{(k-1)k}{2} \alpha}{1 + \frac{k(k+1)}{2} \alpha} \approx P_0 \left(1 - \frac{(k-1)k}{2} \alpha\right) \left(1 - \frac{k(k+1)}{2} \alpha\right) \cong P_0 (1 - k^2 \alpha)$$

and substituting for  $\alpha$  we obtain the relative state probabilities or:

$$P_k \cong P_0 \left(1 - \sqrt{\frac{2}{\pi}} \frac{\Delta}{\sigma_i} k^2\right) \quad (4)$$

Since the state  $P_k$  has an output value  $\Delta k$ , we can write the probabilities of the output values as

$$P(\Delta k) = P(0) \left(1 - \sqrt{\frac{2}{\pi}} \frac{\Delta}{\sigma_i} k^2\right)$$

Finally letting  $x_0$  equal  $\Delta k$  we have the probability of the output value  $x_0$  where  $x_0$  is restricted to integral multiples of  $\Delta$ .

$$P(x_0) = P(0) \left(1 - \sqrt{\frac{2}{\pi}} \frac{x_0^2}{\Delta \sigma_i}\right) \quad (5)$$

Although equation (5) has a parabolic form, it has been obtained from truncation of a Taylor series, and it would be reasonable to suspect that the true form of  $P(x_0)$  is approximately Gaussian. This is also borne out by experiment and by computation of  $P(x_0)$  directly from equation (1). Using this assumption, we may compare equation (5) with the Taylor expansion of the Gaussian distribution:

$$P(x_o) = \frac{1}{\sqrt{2\pi} \sigma_o} e^{-\frac{x_o^2}{2\sigma_o^2}} \cong \frac{1}{\sqrt{2\pi} \sigma_o} \left(1 - \frac{x_o^2}{2\sigma_o^2} + \dots\right)$$

Equating the coefficients of the squared terms in the series expansion we obtain:

$$\frac{1}{2\sigma_o^2} = \sqrt{\frac{2}{\pi}} \frac{1}{\Delta\sigma_i} \quad \text{or} \quad \sigma_o^2 = \sqrt{\frac{\pi}{8}} \Delta\sigma_i$$

Thus the output standard deviation  $\sigma_o$  of the filter has been determined in terms of the input standard deviation  $\sigma_i$  and the step size  $\Delta$  as:

$$\sigma_o = \sqrt{\frac{4\pi}{8}} \sqrt{\Delta} \sqrt{\sigma_i} \quad (6)$$

The result is valid for cases where the step size  $\Delta$  is small compared to  $\sigma_i$ .

Notice that the output standard deviation is proportional to the square root of the input standard deviation. This is the striking feature of this filter, since linear filters would have a direct proportionality between input and output standard deviations. The significance of this effect will be discussed later in this report.

Aside from the noise output of the filter, the output also differs from the ideal input in that it lags behind the input whenever the input has some rate of change,  $\rho$  units per second. This occurs because the output must be offset from the center of the input density function in order for the probability of moving in the direction of input motion to be significantly greater than the probability of moving in the opposite direction. In particular, if the time interval between consecutive decisions is  $T$  seconds and the rate is  $\rho$ , the probability  $P$  of moving in the direction of input motion must be

$$P = \frac{1}{2} + \frac{1}{2} \frac{\rho}{\Delta} T \quad (7)$$

in order that the probability of moving in the proper direction will exceed the probability of moving in the opposite direction by  $\frac{\rho}{\Delta} T$  and the net rate of output motion will match the rate of input motion  $\rho$ . The lag must be such that the areas left and right of the steady state point on the input noise density function produce the above probability. For small lags  $\ell$  and a standard deviation of the normally distributed input signal  $\sigma_i$ , the probability of moving in the proper direction (opposite the lag) is

$$P = \frac{1}{2} + \frac{l}{\sqrt{2\pi} \sigma_i} \quad (8)$$

Equating equations (7) and (8) we have

$$P - \frac{1}{2} = \frac{1}{2} \frac{\rho}{\Delta} T = \frac{l}{\sqrt{2\pi} \sigma_i}$$

Hence the lag is related to the input rate and standard deviation and the step size by

$$l = \sqrt{\frac{\pi}{2}} \frac{\rho}{\Delta} \sigma_i T \quad (9)$$

This represents a constant bias error due to constant rate of changes  $\rho$  in the input signal.

#### FILTER WITH RATE CORRECTION

The problem of steady state lag may be eliminated by having the filter automatically correct itself for the observed rate of change of either input or output. This may be done by measuring the relative probabilities of increase and decrease decisions required to follow the input signal, and using this measurement to make additional corrections in the output rate to compensate for the lag. The particular solution chosen in the tracker<sup>2</sup> was to store a rate estimate as well as a position estimate in the tracker and use the same error signals to change both estimates. Thus an observed error in the negative direction will increase the position estimate by an amount  $\Delta$  and increase the rate estimate by an amount  $\theta$ . This rate estimate is inserted in the position estimate along with the original position decisions. Thus in the steady state with the input changing at a constant rate, the rate estimate must stay constant and there must be as many increase as decrease decisions going into both rate and position estimates. This means that  $P$  must be  $\frac{1}{2}$ , and by equation (8) the steady state lag must be zero.

The filter with the rate correction added may be analyzed by drawing a logical diagram of the filter as shown in Figure 1 and converting it to an equivalent linear system. The only nonlinear part of the system is the clipper and for frequencies low compared to the sampling frequency we can approximate it with a linear component as follows:

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<sup>2</sup>(A rate-compensated tracker is presently being developed.)

Assume we have a clipper with an input quasi-static signal  $x_s$  and an input noise  $x_n$  with a standard deviation  $\sigma_i$ . Also assume the output of the clipper is plus one if the input is positive and minus one if the input is negative. If  $P(-)$  is the probability that the input  $x_n + x_s$  is negative (and thus that the output is negative), then the average output will be given by  $1 - 2P(-)$ . But if the distribution function of the noise is  $F(x)$ , then  $P(-)$  is the probability that  $x_n \leq -x_s$  or  $P(-) = F(-x_s)$ . Thus the average output becomes  $1 - 2F(-x_s)$ . For a Gaussian noise input of standard deviation  $\sigma_i$ ,

$$F(x) \approx \frac{1}{2} + \frac{x}{\sqrt{2\pi} \sigma_i}, \text{ and the average output becomes } x_o = \sqrt{\frac{2}{\pi}} \frac{x_s}{\sigma_i}.$$

Thus for low frequencies and for signals small compared to the noise, the clipper may be replaced by an amplifier whose gain is given by  $G = \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_i}$ . For frequencies low compared to the

sampling frequency the summers may also be replaced by integrators with gains appropriate to the sampling frequency and the samplers removed. The resulting system is shown in Figure 2 and is linear and continuous in both amplitude and time.

The response characteristics of the filter may now be determined in the Laplace transform domain simply by replacing the integrators by  $1/s$ . The forward gain of the system becomes  $\frac{G}{Ts} (\Delta + \frac{\theta}{s})$  and the closed loop transfer function is:

$$H(s) = \frac{\frac{G}{Ts} (\Delta + \frac{\theta}{s})}{1 + \frac{G}{Ts} (\Delta + \frac{\theta}{s})} = \frac{G\Delta s + G\theta}{Ts^2 + G\Delta s + G\theta} \quad (10)$$

It may be seen that the frequency response of the filter is that of a damped second order system where the resonant frequency is determined by the rate step size  $\theta$  and the damping is due to the position steps  $\Delta$ . However the gain  $G$ , which is a function of the input standard deviation, is a factor in both resonant frequency and damping ratio. Thus the parameters  $\Delta$  and  $\theta$  must be chosen to satisfy the desired stability and frequency response requirements for all anticipated values of input standard deviation. In particular for critical damping we must require that:

$$\frac{\Delta}{\theta} \geq \sqrt{8\pi} \sigma_i T \quad (11)$$

at the largest anticipated value of  $\sigma_i$ , where  $G$  has been replaced by  $\sqrt{\frac{2}{\pi}} \frac{1}{\sigma_i}$ .

# COMPARISON OF LINEAR AND NONLINEAR FILTER PERFORMANCE

An index which may be used to express the performance of a nonlinear filter is the equivalent integration time of the filter compared to a simple linear filter. This integration time is defined as the time constant of a simple RC low-pass filter which would give equivalent performance.

The most direct approach for finding the equivalent integration time comes from the transfer function derived for the filter in equation (10). For the case of the filter without rate correction (that is  $\theta = 0$ ), the transfer function of the filter would be

$$H(s) = \frac{1}{\frac{Ts}{G} + 1}$$

We may compare this with the transfer function of a simple RC integrator with time constant  $\tau$ , which is:

$$H(s) = \frac{1}{\tau s + 1}$$

Comparing like terms in the two equations, and remembering that  $G = \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_1}$ , we see that the small-signal transfer function of the nonlinear filter is equivalent to that of an RC integrator with a time constant given by:

$$\tau = \sqrt{\frac{\pi}{2}} \frac{\sigma_1}{\Delta} T \quad (12)$$

An independent check on this result may be obtained from equation (9) for the lag due to a constant rate of change of the input. It may be shown that a simple RC integrator has a lag given by  $\rho\tau$ . Substituting this for the lag shown in equation (9) and solving for the  $\tau$  which will give equivalent lags, we obtain the same result given in equation (12).

It is possible at this point to compare the noise output given in equation (6) with the noise output from the RC integrator with equivalent time constant  $\tau$ . For the RC filter the output at a given time  $t$  is given by:

$$X_0(t) = (1 - e^{-\frac{t}{\tau}}) \sum_{n=0}^{\infty} e^{-\frac{nT}{\tau}} X_1(t - nT) \quad (13)$$

If we assume that each input sample has a variance  $\sigma_i^2$ , the output variance may be computed as the sum of the weighted input variances, assuming the input samples are independent (white noise). Thus:

$$\begin{aligned}\sigma_o^2 &= (1 - e^{-\frac{T}{\tau}})^2 \sum_{n=0}^{\infty} (e^{-n\frac{T}{\tau}})^2 \sigma_i^2 \\ &= (1 - e^{-\frac{T}{\tau}})^2 \sigma_i^2 \sum_{n=0}^{\infty} e^{-2n\frac{T}{\tau}} \\ &= \sigma_i^2 (1 - e^{-\frac{T}{\tau}})^2 \frac{1}{1 - e^{-2\frac{T}{\tau}}}\end{aligned}\quad (14)$$

For sufficiently large  $\frac{T}{\tau}$ , this reduces to:

$$\sigma_o^2 \cong \sigma_i^2 \frac{(\frac{T}{\tau})^2}{2\tau} = \sigma_i^2 \frac{T}{2\tau}\quad (15)$$

$$\text{or } \sigma_o = \sigma_i \sqrt{\frac{T}{2\tau}}\quad (16)$$

Substituting  $\tau$  from equation (12) into equation (16) we obtain the output standard deviation from the equivalent linear filter as:

$$\sigma_o = \frac{1}{4\sqrt{2\pi}} \sqrt{\Delta} \sqrt{\sigma_i}\quad (17)$$

Comparing this to equation (6), we find that the noise output of the nonlinear filter is greater than that of the linear filter with equivalent transfer function by  $\sqrt{\frac{\pi}{2}}$ .

Another method of expressing the performance of the nonlinear filter compared to that of a linear filter is to equate the output standard deviations in equations (6) and (16), and solve for the time constant of the linear filter that would have equivalent output noise. The resultant time constant is:

$$\tau = \sqrt{\frac{2}{\pi}} \frac{\sigma_i}{\Delta} T\quad (18)$$

which is less than equation (12) by  $\frac{2}{\pi}$ . That is, for equivalent output noise the nonlinear filter has a larger effective time constant, and thus less bandwidth, than the linear filter.

It is interesting to note in the case where the output standard deviations of the two filters are equated that the step size  $\Delta$  used in the nonlinear filter is exactly equal to the expected magnitude of the steps in the linear system, which is given by

$\sqrt{\frac{2}{\pi}} \frac{I}{\tau} \sigma_i$ . Throwing away the magnitude and using only the polarity information in the error signal results in a decrease in bandwidth, or if the step size is adjusted for equal bandwidth the output noise is increased.

#### EXPERIMENTAL VERIFICATION

In order to provide an experimental check on the analytical results for the filter, a program was written for the IBM 704 computer which simulated the tracker operation and measured the output standard deviation as a function of the step size  $\Delta$ . A constant input standard deviation of unity was used in the experiment. The results of the experiment, covering step sizes from 0.1 to 0.001, are shown in Figure 3 along with the theoretical result. The experimental standard deviation was consistently 11% to 15% above the expected value. This may be due to non-independence of the input samples (the input was a list of 10,000 numbers generated by a random number routine) or may be a result of the assumptions made in deriving equation (6). It is clear from Figure 3, however, that the form of equation (6) is correct and differs at most by a small multiplicative constant from the correct result. Experiments run on the PUFFS WOX-1A Tracker have verified equations (6) and (9) within the accuracy to which the input standard deviation is known (in fact equation (9) is used to measure the standard deviation of the input to the tracker), and equations (10) and (11) have been verified through experiments on the rate correction circuits for the tracker.

#### DISCUSSION AND CONCLUSIONS

The analytical work done on this nonlinear filter shows that the main feature distinguishing it from a linear filter is the variation of its effective integration time with input standard deviation. Thus as the input noise increases the effective integration time is increased, with the result that the output noise only increases as the square root of the input noise. System frequency response is thus maintained as long as the input signal is good enough to permit it, and as the input noise increases bandwidth is sacrificed to maintain a reasonable output noise level. The most direct effect of the loss of bandwidth is increased lag, but as we have seen it is possible to provide rate correction to eliminate this problem. The filter

may then be considered as an adaptive system capable of adjusting its bandwidth automatically to reach some sort of happy compromise between bandwidth and output noise as the input noise changes. The price paid for this adaptive feature is a slight increase (about 23%) in the output noise of the system if the dynamic response is kept the same, or a reduction of the bandwidth of the system by a factor of 0.66 if output noise is maintained at the same level.

This sort of filter is particularly adaptable to the PUFFS tracker and to other tracking or post filtering problems in sonar or radar systems, and to filtering problems in navigation systems. Typically in these problems, as the range to the target of the navigational facility increases the noise in the raw data increases, but at the same time the rates of change of input data decrease and the required bandwidth is thereby reduced. Thus a filter such as the one described makes use of the lower bandwidth requirement to provide additional smoothing of the raw data and considerably extend the operating range of the entire system. Use of this concept in the WOX-1A PUFFS Tracker has proved the usefulness of such filters by considerably increasing the tracking capabilities over previous linear systems.



## LIST OF SYMBOLS

<u>Symbol</u>	<u>Use</u>	<u>Dimensions</u>
$X_1$	Input Variable	units (of input variable)
$X_o$	Output variable	units
$X_s$	Signal Component of Input	units
$X_n$	Noise Component of Input	units
$\sigma_i$	Input Standard Deviation	units
$\sigma_o$	Output Standard Deviation	units
$\Delta$	Position Step Size	units
$\theta$	Rate Step Size	units/second
$k$	Output State Number	dimensionless
$F(x)$	Cumulative Distribution Function	dimensionless
$P_k$ or $P(x)$	Probability of Output State or Value	dimensionless
$\rho$	Rate of Change of Input	units/second
$l$	Lag of Output behind Input	units
$G$	Gain of Clipper Analogue	$\sqrt{\frac{2}{\pi}} \frac{1}{\sigma_i}$ units <sup>-1</sup>
$s$	Laplace Operator	seconds <sup>-1</sup>
$H(s)$	Transfer Function of Filter	dimensionless
$\tau$	Effective Integration Time (Exponential Time Constant)	seconds
$T$	Sampling (Decision) Period	seconds

FIG. 1 LOGICAL DIAGRAM OF FILTER

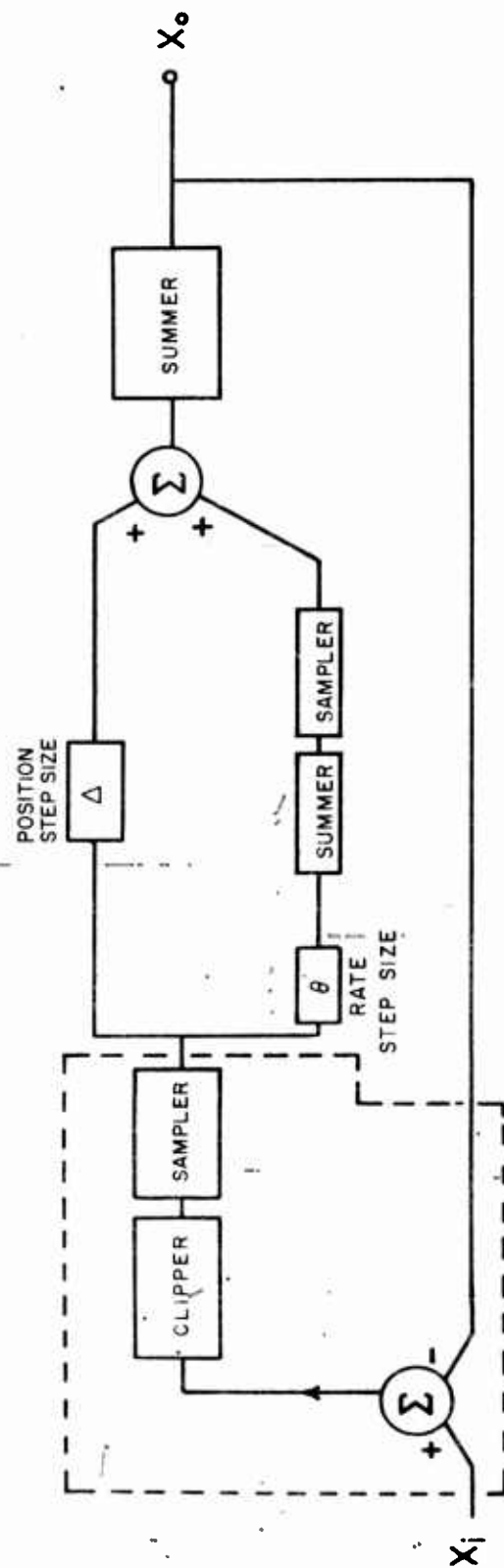


FIG. 2 LINEARIZED CONTINUOUS ANALOGUE OF FILTER

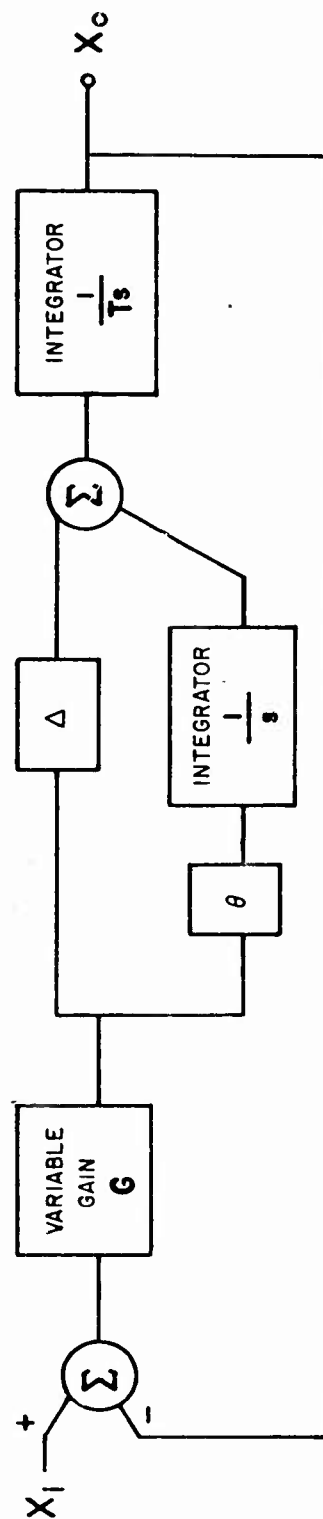
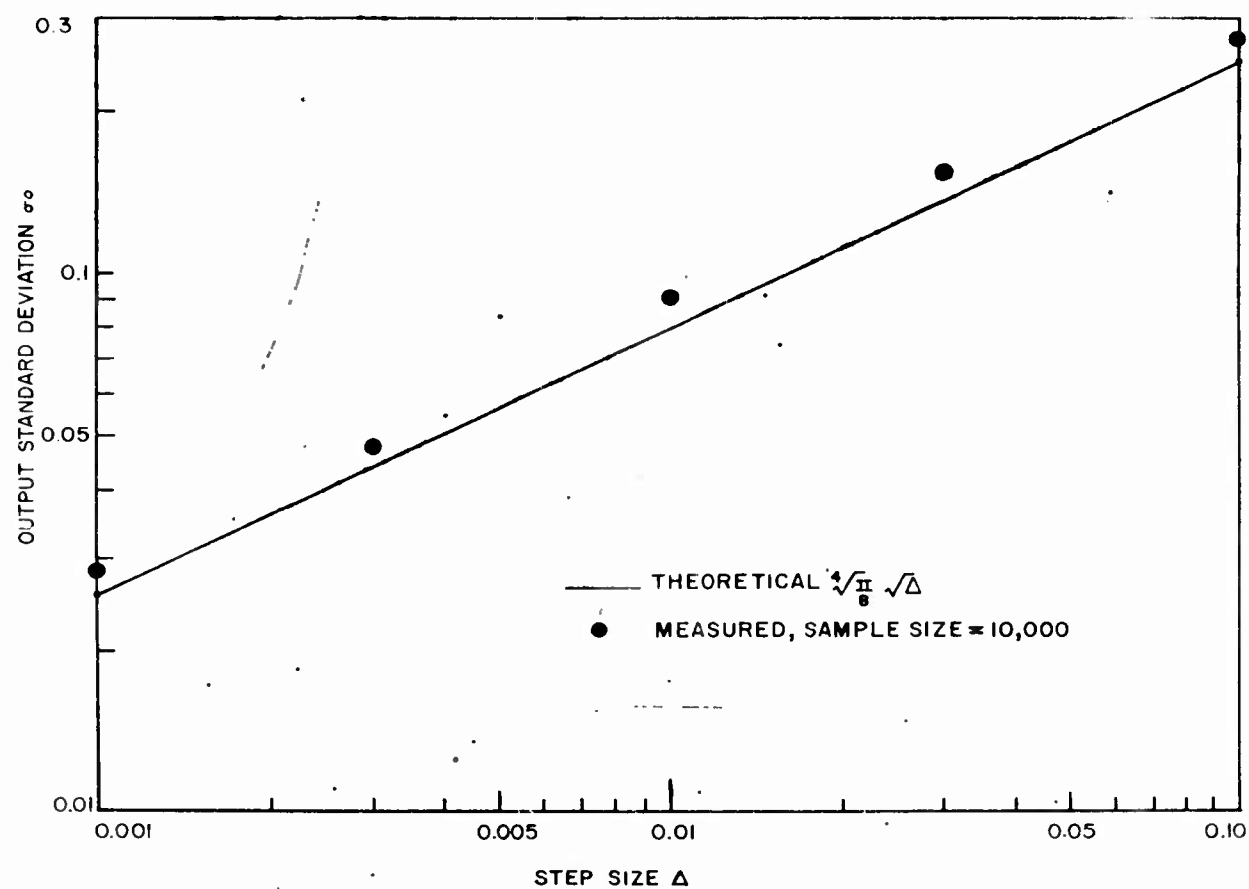


FIG.3 OUTPUT STANDARD DEVIATION VS. STEP SIZE

NOL-IBM 704 SIMULATION 25 NOV. 1960

INPUT STANDARD DEVIATION = 1.00



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